Final Year Project

Calculation of a turbulent Couette Flow using various turbulent models

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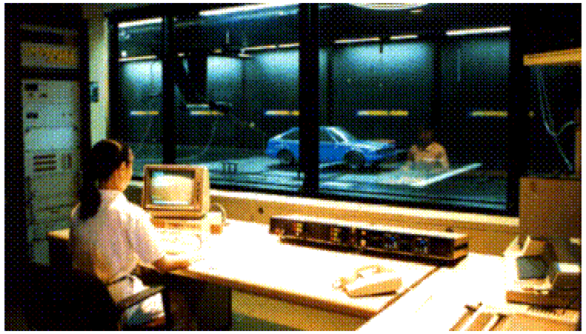
Turbulence modeling

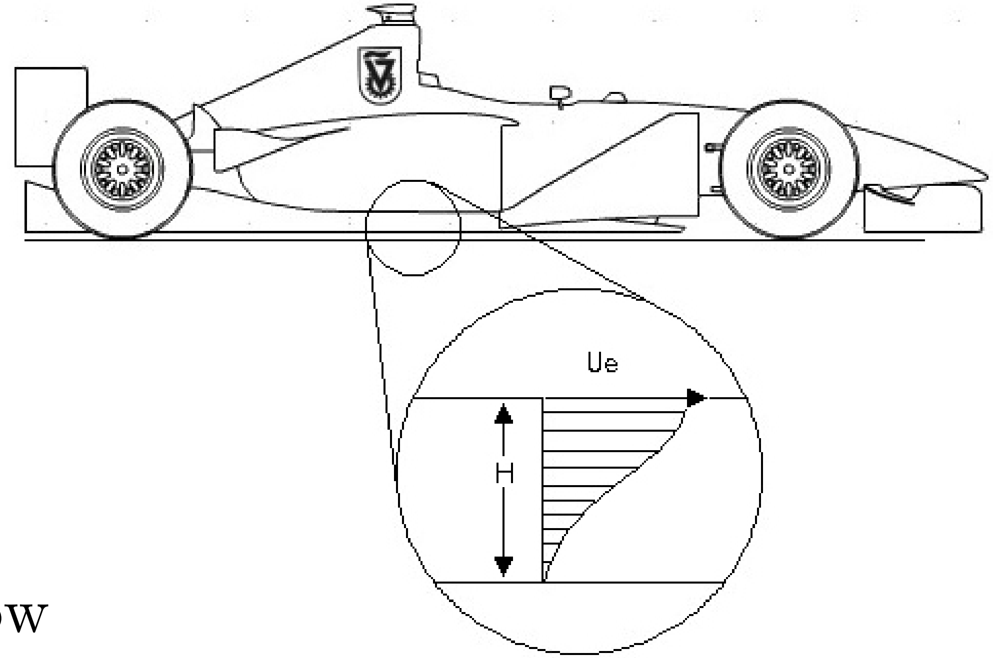
DNS



Experimental

Turbulence modeling

Turbulent Couette Flow



- Channel Flow
- Boundary layer flow

The problem

Solve the velocity profile of a Couette flow using turbulence models

Assumptions:

- It is a 1-D problem
- Incompressible flow
- Steady flow
- Plates are at the same temperature
- Temperature changes are neglected

Defining the + system

$$u^{+} = \frac{u}{u_{\tau}} = \frac{u}{u_{e}}\sqrt{\frac{2}{Cf}}$$

$$y^{+} = \frac{y \cdot u_{\tau}}{\nu} = \mathrm{Re}\frac{y}{H}\gamma$$

Where :

$$u_{\tau} = \sqrt{\frac{T_{w}}{\rho}} \qquad T_{w} = \mu \cdot \left(\frac{\partial u}{\partial y}\right)_{wall}$$

The boundary Layer

1. Viscous sublayer $y+ < 5$
2. Buffer layer $5 < y+ < 30$
3. Logarithmic layer $y+ > 30$, $y < 0.1\delta$
4. Defect layer $0.2 < y/\delta < 1$

The mixing Length model

The two dimensional, steady x-momentum Navier-Stokes

$$\rho \cdot u \cdot \frac{\partial u}{\partial x} + \rho \cdot v \cdot \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left[\mu \cdot \left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}\right)\right]$$

For this case:

$$\frac{\partial u}{\partial x} = 0, \quad \frac{\partial p}{\partial x} = \mathrm{P}, \quad v = 0$$

Therefore

$$\frac{\partial}{\partial y}\left[\left(\nu + \nu_t\right) \cdot \left(\frac{\partial u}{\partial y}\right)\right] = 0$$

The mixing length model , length scale and turbulent viscosity

$$l = \kappa \cdot \hat{y} \cdot \left[1 - e^{\frac{y+}{A}} \right]$$

$$\hat{y} = \min(y, h - y)$$

$$\nu_t = \ell^2 \frac{\partial u}{\partial y}$$

Non dimensional equation form

$\eta = y / H$

$z = u / u_e$

$n = \nu_t / u_e H$

$$\frac{\partial}{\partial \eta}\left[\left(\frac{1}{\mathrm{Re}} + n\right)\frac{\partial z}{\partial \eta}\right] = 0$$

$$n = \kappa^2 \hat{\eta}^2 \left[1 - e^{\frac{-\mathrm{Re}\cdot\hat{\eta}\cdot\gamma}{A}}\right]^2 \left(\partial u / \partial y\right)$$

$$\hat{\eta} = \min(\eta, 1-\eta)$$

Mixing length model Discretisation

$$\frac{a_{i+1/2}\left(\frac{\phi_{i+1}-\phi_{i}}{\Delta y}\right)-a_{i-1/2}\left(\frac{\phi_{i}-\phi_{i-1}}{\Delta y}\right)}{\Delta y}=0$$

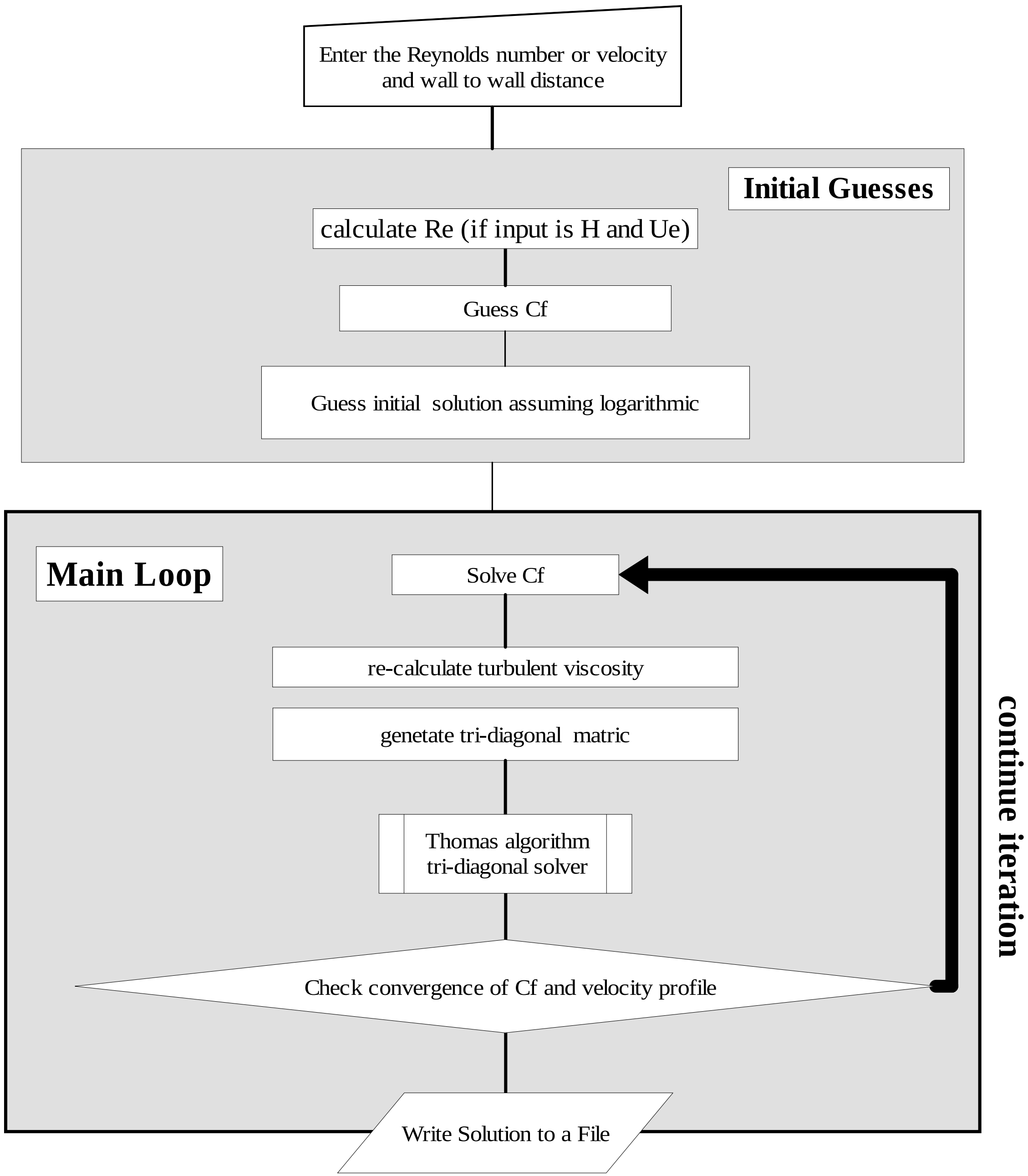
$$a=\frac{1}{\mathrm{Re}}+\kappa^{2}\hat{\eta}^{2}\left[1-e^{\frac{-\mathrm{Re}\cdot\hat{\eta}\cdot\gamma}{A}}\right]^{2}\left(\partial u/\partial y\right)$$

Solving the equation numerically

$$\underbrace{\phi_{i-1}\left(a_{i-1/2}\cdot b_{i-1}\right)}+\underbrace{\phi_i\left(-a_{i+1/2}\cdot b_i-a_{i-1/2}\cdot b_i\right)}+\underbrace{\phi_{i+1}\left(a_{i+1/2}\cdot b_{i+1}\right)}=0$$

A B C

$$\begin{bmatrix} 1 & 0 & & & \\ A & B & C & & \\ & \ddots & \ddots & \ddots & \\ & & A & B & C \\ & & & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} = \begin{bmatrix} boundary \\ r_1 \\ \vdots \\ r_{n-1} \\ boundary \end{bmatrix}$$



Enter the Reynolds number or velocity and wall to wall distance
Initial Guesses
calculate Re (if input is H and Ue)
Guess Cf
Guess initial solution assuming logarithmic
Main Loop
Solve Cf
re-calculate turbulent viscosity
genetate tri-diagonal matric
Thomas algorithm tri-diagonal solver
Check convergence of Cf and velocity profile
continue iteration
Write Solution to a File

K-ε model

- Calculate turbulent Kinetic energy
- Calculate dissipation
- Calculate turbulent viscosity
- Calculate velocity

K-ε model , Kinetic Energy

$$\rho \cdot u \frac{\partial k}{\partial x} + \rho \cdot v \frac{\partial k}{\partial y} = \frac{\partial}{\partial y}\left[\left(\mu + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial y}\right] + P_k - D_k + S_k$$

$$\frac{\partial}{\partial y}\left[\left(\mu + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial y}\right] + P_k - D_k + S_k = 0$$

K-e model Energy Dissipation

$$\rho \cdot u \frac{\partial \varepsilon}{\partial x} + \rho \cdot v \frac{\partial \varepsilon}{\partial y} = \frac{\partial}{\partial y}\left[\left(\mu + \frac{\mu_t}{\sigma_k}\right)\frac{\partial \varepsilon}{\partial y}\right] + P_\varepsilon - D_\varepsilon + S_\varepsilon$$

$$0 = \frac{\partial}{\partial y_j}\left[\left(\mu + \frac{\mu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial y}\right] + P_\varepsilon - D_\varepsilon + S_\varepsilon$$

Terms

$$P_k = \mu_\tau \cdot \left(\frac{\partial u}{\partial y} \right)^2$$

$$P_\varepsilon = C_{\varepsilon 1} \cdot \frac{\varepsilon}{k} \cdot P_k$$

$$D_k = \rho \cdot \varepsilon$$

$$D_\varepsilon = C_{e2} \cdot f2 \cdot \rho \cdot \frac{\varepsilon^2}{k}$$

Turbulent viscosity

$$\mu_t = C_\mu \cdot f_\mu \cdot \rho \frac{k^2}{\varepsilon}$$

Jones and Launder Coefficients

$$f_\mu = e^{\left(\frac{-2.5}{1+Rt/50}\right)}$$

$$f_2 = 1 - 0.3 \cdot e^{-Rt^2}$$

$$\sigma_k S_k = -2\mu \left(\frac{\partial \sqrt{k}}{\partial y}\right)^2$$

$$S_\varepsilon = 2\mu \cdot \mu_t \left(\frac{\partial^2 \overline{u}}{\partial y^2}\right) \quad 1971$$

$$S_\varepsilon = 2\mu \cdot \mu_t \left(\frac{\partial^2 \overline{u}}{\partial y^2}\right)^2 \quad 1972$$

$$\frac{a_{i+1/2}\left(\frac{\phi_{i+1}-\phi_i}{\frac{1}{2}\left(y_{i+1}-y_i\right)}\right)-a_{i-1/2}\left(\frac{\phi_i-\phi_{i-1}}{\frac{1}{2}\left(y_i-y_{i-1}\right)}\right)}{\frac{1}{2}y_{i+1}-y_{i-1}}+P-D+S=0$$

k

$$A=\left[\frac{\phi_{i-1}\left(a_{i-1/2}\right)}{\frac{1}{4}dy^2}\right] \quad B=\left[\frac{\phi_i\left(-a_{i+1/2}-a_{i-1/2}\right)}{\frac{1}{4}dy^2}\right] \quad C=\left[\frac{\phi_{i+1}\left(a_{i+1/2}\right)}{\frac{1}{4}dy^2}\right] \quad R=-\left(P_k-D_k+S_k\right)$$

ε

$$A=\left[\frac{\phi_{i-1}\left(a_{i-1/2}\right)}{\frac{1}{4}dy^2}\right] \quad B=\left[\frac{\phi_i\left(-a_{i+1/2}-a_{i-1/2}\right)}{\frac{1}{4}dy^2}\right] \quad C=\left[\frac{\phi_{i+1}\left(a_{i+1/2}\right)}{\frac{1}{4}dy^2}-D_\varepsilon\right] \quad R=-\left(P_\varepsilon+S_\varepsilon\right)$$

$$a=\left[\mu+\frac{\mu_t}{\sigma_k}\right]$$

The Law of the wall

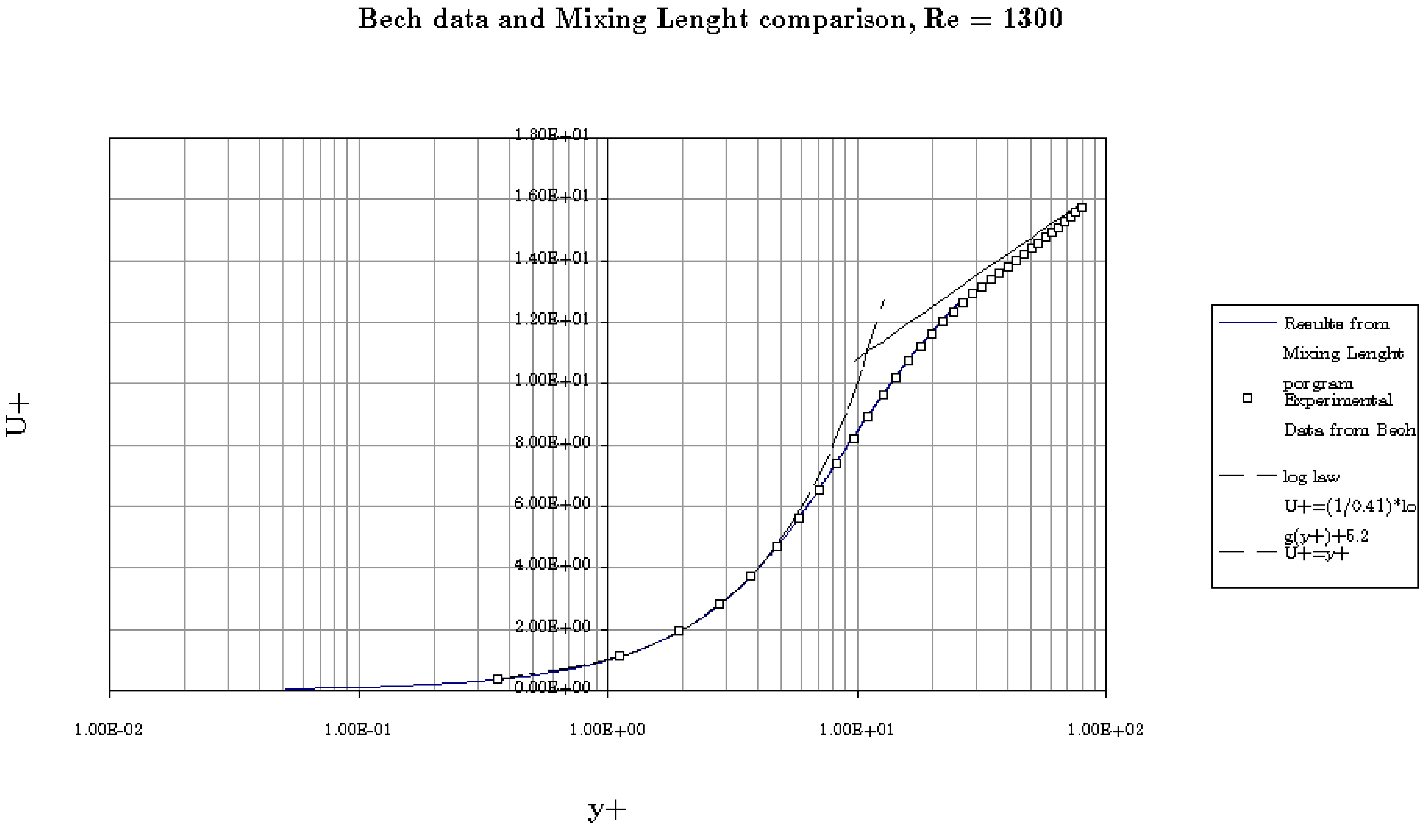
The viscous sub-layer

$$u_+ = y_+$$

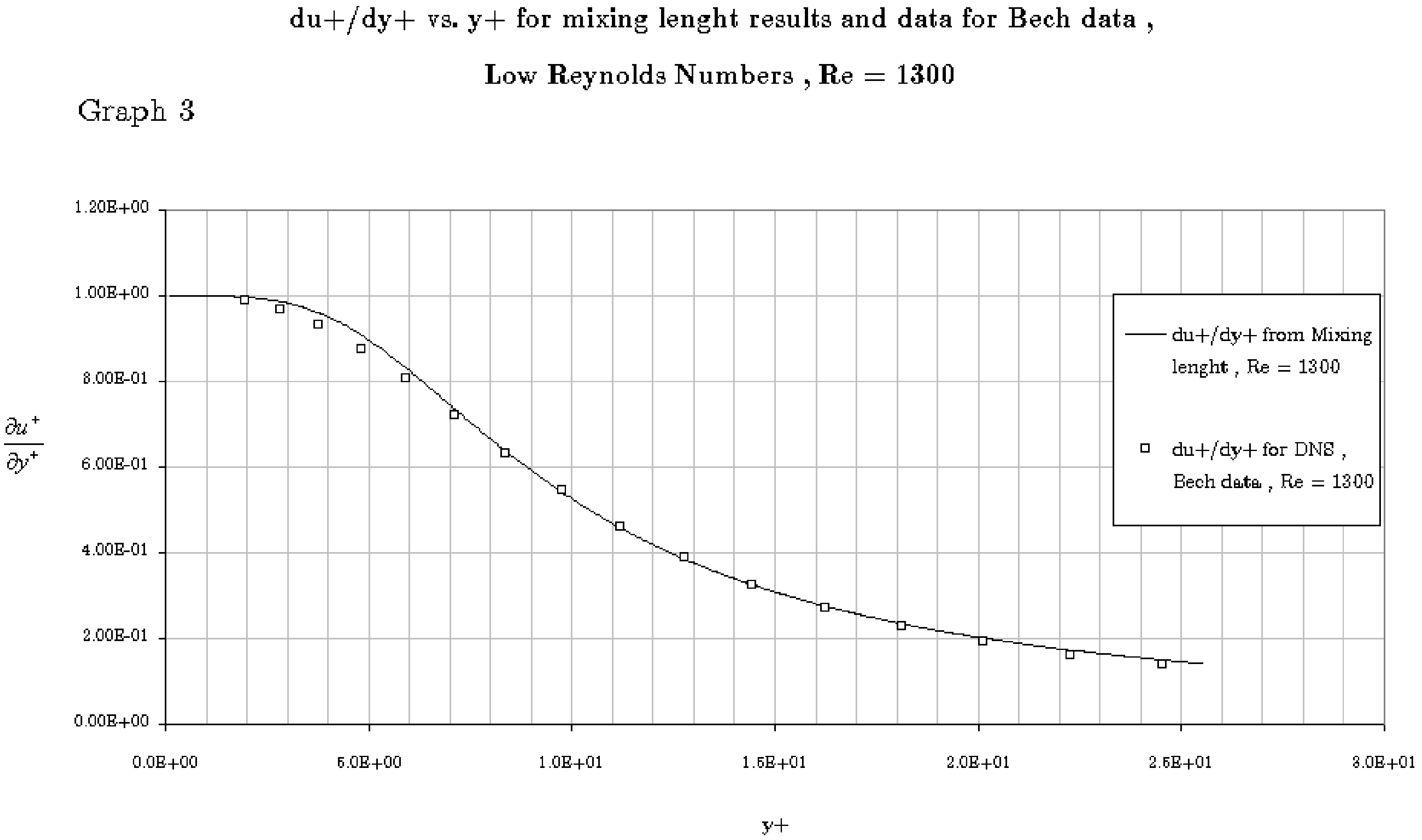
For the logarithmic region

$$u^+ = \frac{1}{\kappa}\log_e y^+ + C$$

Low Reynolds numbers , DNS data

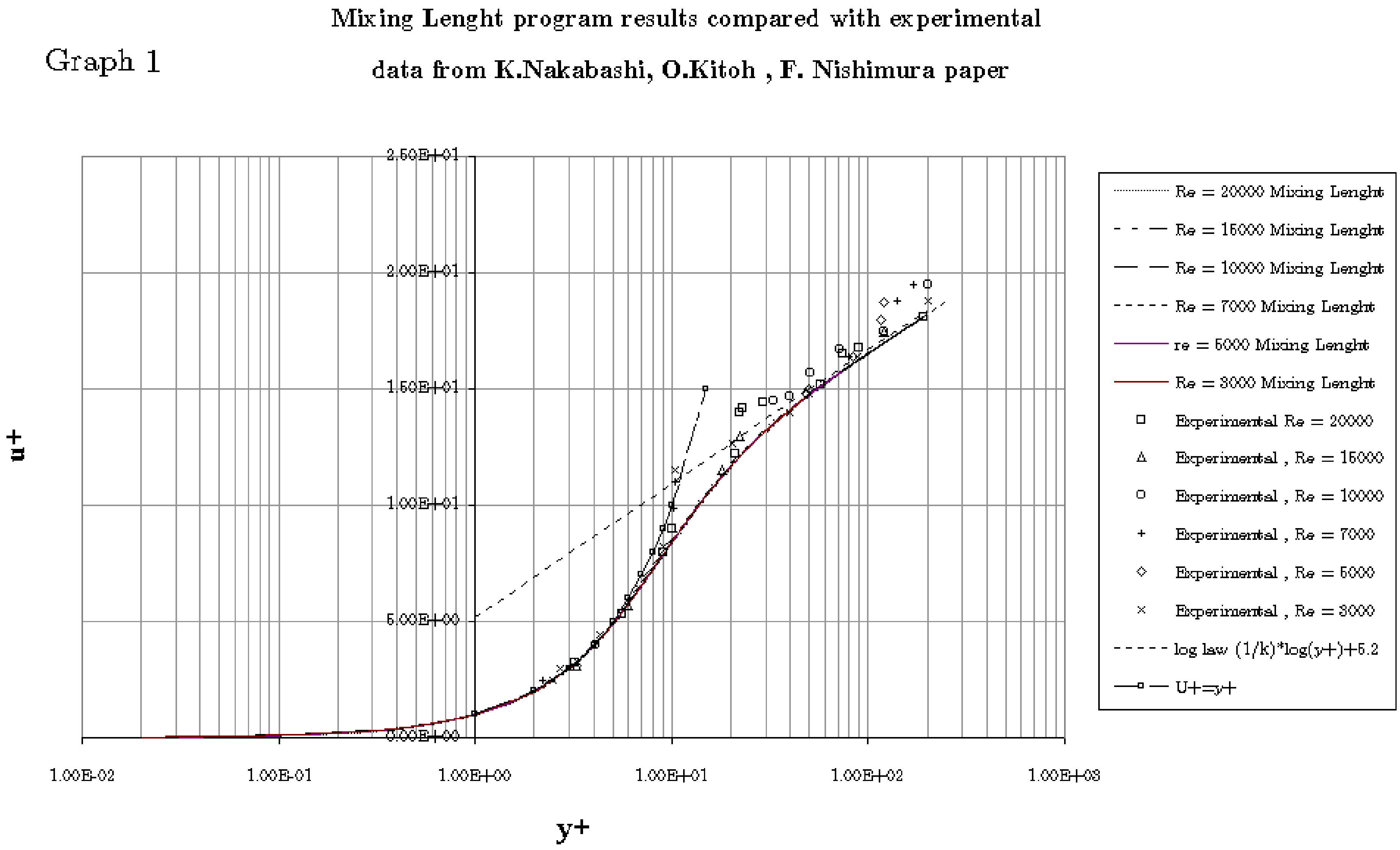


Slope for Low Re. And DNS data



Higher Reynolds numbers , Experimental data

Graph 1



Conclusion

•The mixing length turbulence model can provide a reasonable estimate of the mean velocity for a one dimensional, incompressible turbulent Couette flow for a range of Reynolds numbers up to 20000.

•The assumption of a one-dimensional flow for this problem is valid .

•The law of the wall provides a good initial estimation for the velocity profile for this problem.